Global Math Webinar
October 10, 2017
9:00 pm

Implementing Effective Mathematics Teaching Practices

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Handout Packet

Activity 1

Analyzing Teaching and Learning 3.1

Comparing Two Tasks Involving Proportional Relationships

1. Solve the Candy Jar task and the Finding the Missing Value task. Consider the strategy that you use to solve each task and the features of the problem that led you to use it.
2. Consider the ways in which the two tasks are the same and the ways in which they are different. Which one is more likely to promote reasoning and problem solving? Why?

The Candy Jar Task	The Finding the Missing Value Task
A candy jar contains 5 Jolly Ranchers and 13 jawbreakers. Suppose that you had a new candy jar with the same ratio of Jolly Ranchers to jawbreakers, but it contains 100 Jolly Ranchers. How many jawbreakers would you have? Explain how you know. Adapted from Smith et al. 2005	Find the value of the unknown in each of the proportions shown below. $\frac{5}{2}=\frac{y}{10}$ $\frac{a}{24}=\frac{7}{8}$ $\frac{n}{8}=\frac{3}{12}$ $\frac{30}{6}=\frac{b}{7}$ $\frac{5}{20}=\frac{3}{d}$ $\frac{3}{x}=\frac{4}{28}$

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Activity 2

Analyzing Teaching and Learning 6.1

Considering the Advantages of Different Representations

1. Solve the Cars and Motorcycles task shown below. Think about the representation that you used and how it helped you make sense of the situation.
2. Review the samples of student work shown in figure 6.2. Consider the following:
   - What does each representation tell you about what the student understands about the situation?
   - How do the different representations connect with one another?

The Cars and Motorcycles Task

Diane looked out the window of her math classroom at the teachers' parking lot and said, "There are 13 motorcycles and cars in the lot." Steve looked out the window and said, "I see 42 wheels." The teacher asked, "How many motorcycles and how many cars are in the parking lot?"

Show all your work, and explain your thinking.

This task was adapted from the QUASAR Cognitive Assessment Instrument (Lane 1993).

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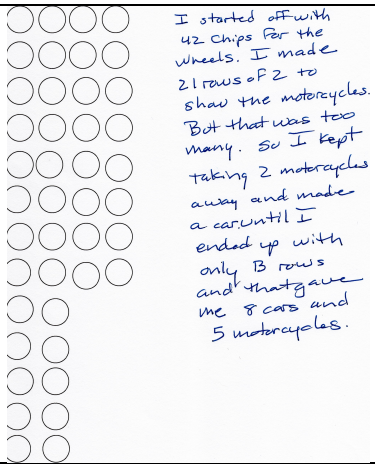
Activity 2
Student Work for the Cars and Motorcycles Task

Student A

$C + M = 13$ C = cars
$4C + 2M = 42$ M = motorcycl
$4(13 - M) + 2M = 42$
$52 - 4M + 2M = 42$
$10 = 2M$
$5 = M$

Number of motorcycles: 5
Number of cars: 8

Student D



I started off with 42 chips for the wheels. I made 21 rows of 2 to show the motorcycles. But that was too many. So I kept taking 2 motorcycles away and made a car until I ended up with only 13 rows and that gave me 8 cars and 5 motorcycles.

Student B

$6 \times 4 = 24$
$7 \times 2 = 14$
38

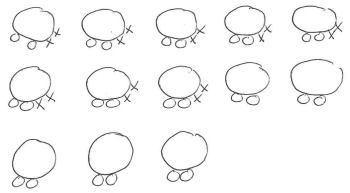
$7 \times 4 = 28$
$6 \times 2 = 12$
40

$8 \times 4 = 32$
$5 \times 2 = 10$
42

Number of motorcycles: 5
Number of cars: 8

Student E

13 CARS AND MOTORCYCLES



EACH HAS 2 WHEELS USING UP 26 WHEELS
THERE ARE 16 WHEELS LEFT SO I PUT
2 MORE WHEELS ON EACH OF 8 CIRCLES

Number of motorcycles: 5
Number of cars: 8

Student C

M	TOTAL WHEELS	C
1	50	12
2	48	11
3	46	10
4	44	9
5	42	8
6		7
7		6
8		5
9		4
10		3
11		2
12		1

Number of motorcycles: 5
Number of cars: 8

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Activity 3

Analyzing Teaching and Learning 7.1 more4U

Analyzing Whole-Class Classroom Discourse

Watch the video clip of the discussion of the Counting Cubes task in Peter Dubno's classroom. While you watch the video, pay attention to the student discourse and the connections that students make between representations. Specifically—

1. What does the discourse reveal about students' understandings of the connections between the pictorial and algebraic representations?
2. To what extent does the discourse facilitate students' explanations or clarifications of their thinking?
3. To what extent does the discourse make mathematics more visible and accessible for student examination and discussion?

You can access and download the videos and their transcripts by visiting NCTM's More4U website (nctm.org/more4u). The access code can be found on the title page of this book.

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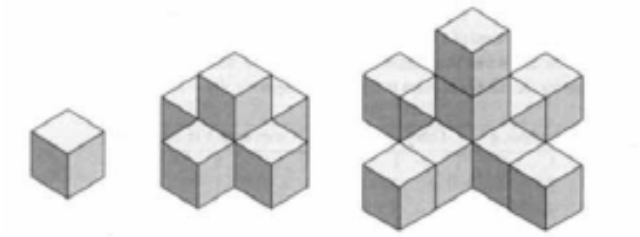
The Counting Cubes Task



1 2 3

1. Describe a pattern that you see in the cube buildings.
2. Use your pattern to write an expression for the number of cubes in the *n*th building.
3. Use your expression to find the number of cubes in the fifth building. Check your results by constructing the fifth building and counting the cubes.
4. Look for a different pattern in the buildings. Describe the pattern, and use it to write a different expression for the number of cubes in the *n*th building.

Adapted from Lappan et al. (2004a).

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Counting Cubes Task

Teacher: Peter Dubno
District: New York City School District
Grade: 8

Teacher: --come up and present what they came up with. Arden, come on. Now remember, one person speaks because we can't hear many people speaking at once.

Student: Okay, everybody, like, on the things there were arms, right, which are the extended parts, right? So our pattern was that every time you add one...a cube to each arm, the total volume increases by 5 cubic centimeters, which I think was 5 cubes or whatever you called it. And then on our whatever...

Student: Equation.

Student: --equation, was that $5n + 1$ equals the volume and n equals the length of one individual arm. So that, like, there's the middle cube, excluding the middle cube, you would multiply that by 5 because there are 5 arms and then add 1 for the middle cube and that will give you the volume and number of cubes.

Student: For the 3rd question, it was, what's the volume of the 5th stage? Well, for the 5th stage, there will be 4 cubes on each arm so the equation will be 5 x 4, + 1 equals the volume. So 5 x 4 is 20, + 1 is 21, so the volume will be 21.

Student: Yeah.

Teacher: Any questions for the presenters? Did anyone come up with a different solution? All right, Cassie come on. Do you want to show us? Cassie, Deirdre?

Student: Originally, we got what Arden got, but we tried it out and it didn't always work.

Student: With the 1st one.

Student: With the first one, so what happened is we came up with $5n - 4$, so it's 5 arms and then n would be the building number.

Student: Yeah, and then you subtract it.

29	*Student:*	And then you subtract 4 and then you get the number of—
30	*Student:*	Cubes in the building.
31 32	*Student:*	--cubes in the building. So we wrote up an example and $n = 2$, so then 5 x 2 - 4 is 10 – 4, and we got 6 cubes in the building.
33	*Student:*	And that's the number of the cubes in the building.
34	*Student:*	And it's on the sheet also.
35 36	*Teacher:*	So let me ask you. Of that formula, if I asked you how many cubes would be in the 7^{th} building—
37	*Student:*	It would be 31.
38	*Teacher:*	And how would you get that?
39 40	*Student:*	You'd replace *n* with the 7 and then you do the work, you'd multiply 7 times 5 and then you get 35, and the you minus 4 and you get 31.
41 42	*Teacher:*	Okay. How is yours different or the same as what Arden did and Yoshio did?
43	*Student:*	Both of us used 5*n*, 5 times the building number for each arm but—
44	*Student:*	The only thing that was different was that we subtracted and he added.
45	*Teacher:*	Does that make it different or is it the same or what?
46 47 48	*Student:*	No...No, we did the middle square for each arm and then you subtracted 4 middle squares. But what we did is we just added the actual length of the arm excluding the middle square.
49 50 51 52	*Student:*	But the reason why it didn't work for us was because we were trying to figure out an equation that would fit all buildings instead of just each every 1 except for the 1^{st} one, because it doesn't really work for the 1^{st} one.
53	*Student:*	Yea. Because there's only 1 cube.
54	*Student:*	Yeah, there's no—
55	*Student:*	5 x 0 = 0, + 1 = 1.
56	*Student:*	It's the 1^{st} building, though.

Student: I have a question. What's 5 x 1? Ok, I think...Wait 1 second. I think what Arden is trying to...He defined *n* as the length of one arm, so for the 1st building it would be 0, not 1. And you were trying to do the building number. So that's what was different about it. That's why you're having this quarrel.

Student: We did that while we were defining it as just the 1st building number was the cube, so we were doing the building. Okay.

Teacher: What about this one? How does this one fit in with that? Is that...there a mathematical equivalence there somehow? Yoshio, do you think you can show us that or explain it or how is it different, how is it the same?

Student: Our definition of *n* is different from theirs. Ours is *n* equals the length of each arm. So the equation will change, will be different from the two.

Teacher: So what you're telling me is the definition of the variable is a very important idea in mathematics?

Students: Yea.

Teacher: Okay, it makes the whole difference of what the expression is?

Student: Yeah, because in there, they're multiplying the—they're considering each arm what we were considering plus the middle and then they were going to subtract 4 middles because they would have 4 extra middles. And that...But what we were doing is we were just multiplying each of the arms without the middle and then adding one middle. It's really just the same thing. It just depends on how you think of it.

[End of Audio]